



## **Performance analysis of the FB-PLL applied to real-time monitoring of deep sleep brainwaves**

### **Análise de desempenho do FB-PLL aplicado ao monitoramento em tempo real de ondas cerebrais de sono profundo**

**Igor Ribeiro Pereira**

Centro Federal de Educação Tecnológica de Minas Gerais

E-mail: igoribeiro.275@gmail.com

**Cláudio Henrique Gomes Dos Santos**

Centro Federal de Educação Tecnológica de Minas Gerais

E-mail: claudiohgsantos@gmail.com

**Keywords:** Deep sleep, FB-PLL, Brainwaves.

#### **1 INTRODUCTION**

Sleep can be considered a universal phenomenon, being present in animals and humans. It occurs naturally, classified as a physiological need not able to be avoided. In its process there is a decrease in body function and consciousness, and despite the existence of visual stimulus or hearing, the person does not show any feedback or response to them (ABEL, 2013).

Based on this, low-frequency wave sleep acts mainly on improving cognition, which consists of the psychological function linked to learning, intellectual and emotional development. In addition, it promotes memory consolidation, as well as acts on the development of children's motor skills (PIEKARSKI, 2015). Finally, it is worth mentioning that this phenomenon directly helps to maintain and promote physical and mental health. Thus, it becomes evident that techniques are needed to monitor and track the behavior of waves present during sleep, as well as referring to its cycle as a whole.

The Fast Fourier Transform (FFT) technique can be used to obtain the characteristics of a signal, such as its frequency. Thus, it can be applied to sleep brain waves. However, to obtain an accurate result using the FFT, a considerable number of sampling cycles is required for its execution. Although, with the addition of so many cycles, considerable time is required to calculate the FFT, making it difficult to apply it for a real-time reading. To perform the tracking of acoustic waves, it is proposed to apply the PLL technique based on Fourier (FB-PLL) to a signal with typical behavior of deep



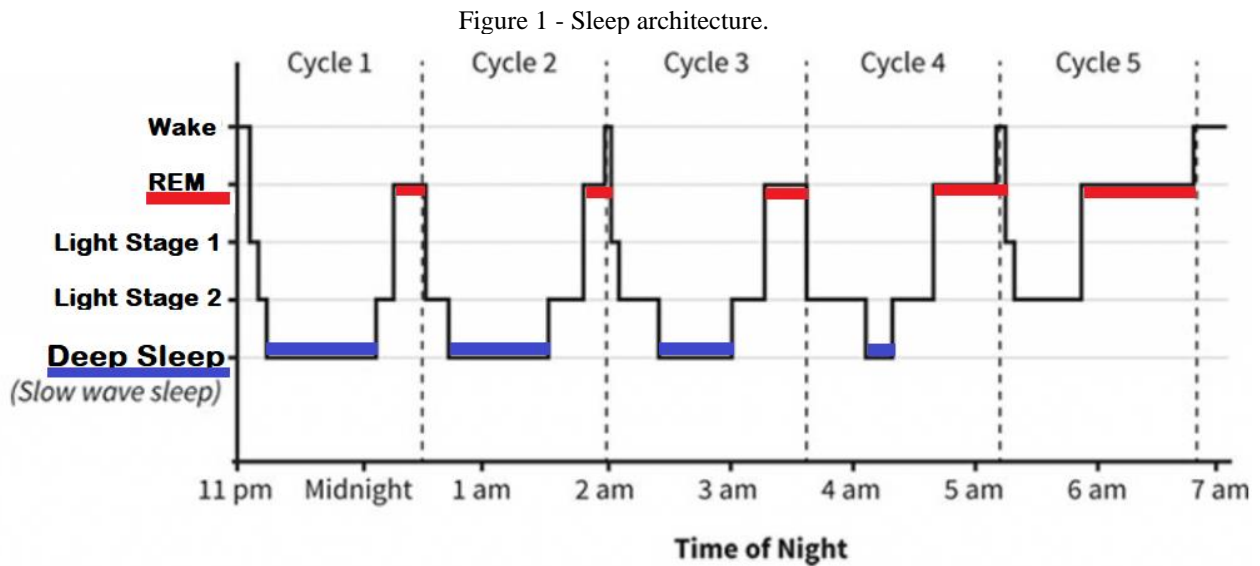
sleep. In this sense, this method is capable of estimating the main frequency, as well as displaying the amplitude and reconstructing the signal in just a single cycle. Which, consequently, makes it possible to obtain a considerably faster response, making this approach feasible for application in real-time monitoring.

Finally, this work is organized in the following order: in Section 2 there is a brief review on the study of sleep, especially regarding Slow Wave activity. In sequence, in Section 3 there is a contextualization about the estimation techniques based on Fourier, focusing on the FFT and the FB-PPL. Later, in Section 4, the results obtained with the application of the FB-PLL are presented. In Section 5, there is the conclusion of the work carried out. Finally, in Section 6, you will find the acknowledgments attributed.

## **2 MAIN ASPECTS RELEVANT TO SLOW WAVE SLEEP**

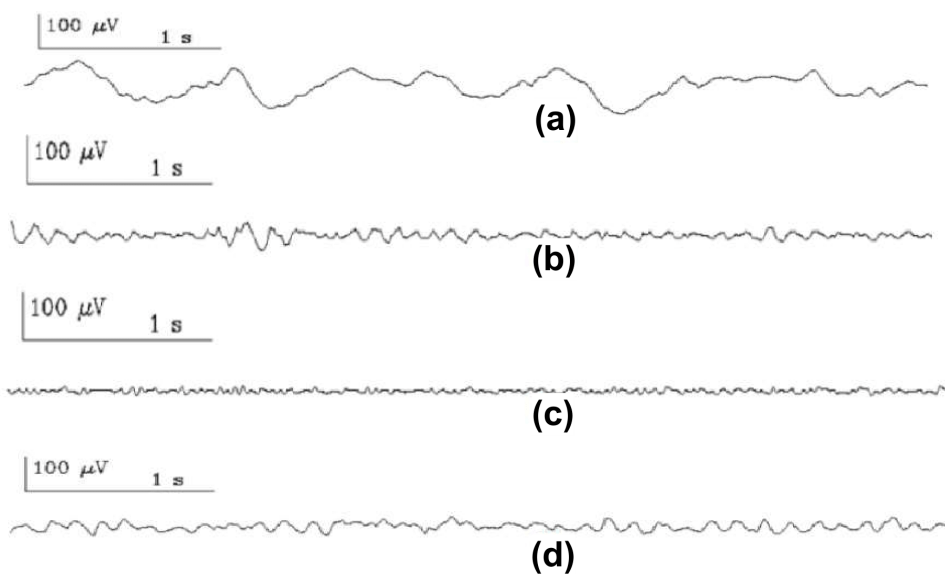
For recording a scientific verification of when a person is sleeping, with the use of electrodes, measurement techniques have been developed, such as the EEG (Electroencephalogram), the EMG (Electromyography) and the EOG (Electrooculogram). Based on this, signals are obtained from three regions, on brain wave activity, eye movement activity and muscle activity, respectively. Therefore, by gathering these signs, a set is formed with the general term PSG, that is Polysomnography, in which it is an exam that contains a complete reading of sleep (PEREIRA, 2023).

One concluded fact is that human beings, when sleeping, go through two types of sleep, namely Rapid Eye Movement Sleep (REM) and Non-Rapid Eye Movement Sleep (NREM). REM sleep is considered the last stage of the cycle, in which brain activity is very similar to the waking state. NREM sleep, on the other hand, is fragmented into three Stages, with Stage 3 considered the deepest sleep that Stages 1 and 2, thus occurring a transition from sleep (WILCKENS). The deepest stage represents Slow Wave Sleep (SWA), which is the focus of this work. From this, through monitoring during the night, the sleep architecture can be obtained, as shown in Figure 1.



The Figure 1 demonstrates the graph technically known as a hypnogram (WALKER, 2009). It is possible to perceive an inequality in the Stages, in which REM and NREM sleep undergo major changes in each ninety-minute cycle throughout the night. In addition, deep NREM sleep is very present in the first half of the night. Then, as you move into the second half of the night, a shift occurs, where REM sleep begins to take up most of it, and deep NREM sleep barely appears. Through the PSG exam, it is also possible to capture and analyze the main waves during sleep, which are called Delta, Theta, Alpha and beta waves, as can be seen in Figure 2.

Figure 2 - Representations relevant to the main waves present in sleep. Being in (a) the Delta wave, in (b) the Theta wave, in (c) the Alpha wave and in (d) the Beta wave.





In Figure 2 (a), the representation of the Delta wave is displayed, thus, it is commonly found in Stage 3 of NREM sleep, that is, in the phase referring to deep sleep. In this sense, the Delta activity has frequencies between 0.5 and 4 Hz, as well as having a greater amplitude, as well as being the slowest elaborated by the brain, being able to associate them with tranquility and well-being (PEREIRA, 2023). Therefore, there are indications that they can interfere with other factors, such as helping to regulate rest and body performance, which can even affect empathy and motivation. Finally, this waveform is present in babies and children, and is also found in the fetus within the mother's womb. However, as we age, the generation of Delta waves tends to decrease.

At the moment, currently, several researches are being carried out based on the study of Slow Wave activity. For example, (WILCKENS) conducted a study in which cognition can be improved by improving SWA. It was also found that the progress of high amplitude and low frequency are related beyond cognition, with the development of executive function and memory consolidation. Furthermore, (ASTILL, 2014) describes that the performance of motor skills in children are reflected in the frequency of slow waves. Thus, there is an increase in children's accuracy, making it possible for sleep to improve certain motor skills, usually without any additional treatment. It is noted that physical and mental health, along with well-being, are directly aligned with a good night's sleep. That said, we emphasize the importance of monitoring the sleep cycle in real time, as well as obtaining brain curves, since, through the interpretation of the results, it is possible to contribute directly to people's quality of life, as well as to promote advances in Medicine of Sleep.

### **3 FOURIER BASED ESTIMATING TECHNIQUES**

The Fourier Transform consists of a mathematical tool that performs the transition from the time-varying domain to the frequency domain. This is necessary since, in general, the graphs obtained in the time domain contain an excess of information, since they are influenced by different amplitudes that are repeated in different periods of time. With the domain change, it is possible to filter the signal, avoiding this repetition and overlapping of amplitudes (SPIEGEL, 1974). Thus, the Fourier Transform of a signal  $x(t)$  is defined by:

$$F = \int_{-\infty}^{\infty} x(t) \cdot e^{-i\omega t} dt, \quad (1)$$



in which  $x(t)$ , is a real-time and integrable function, whereas  $F(\omega)$  consists of a complex frequency function. For the Fourier Transform, it is possible to obtain a representation called frequency spectrum, which is the most used way to represent the characteristics of a captured signal. By means of the frequency spectrum, the different frequencies and the magnitude that make up the analyzed signal (SPIEGEL, 1974) can be evaluated. However, obtaining the spectrum through (1) sometimes becomes complicated. From this, an applied solution consists of replacing (1) and using Discrete Transform of Fourier (DFT), which is described by the following expression:

$$F(m) = \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{i2\pi nm}{N}} \quad m = 0, 1, 2 \dots, n - 1, \quad (2)$$

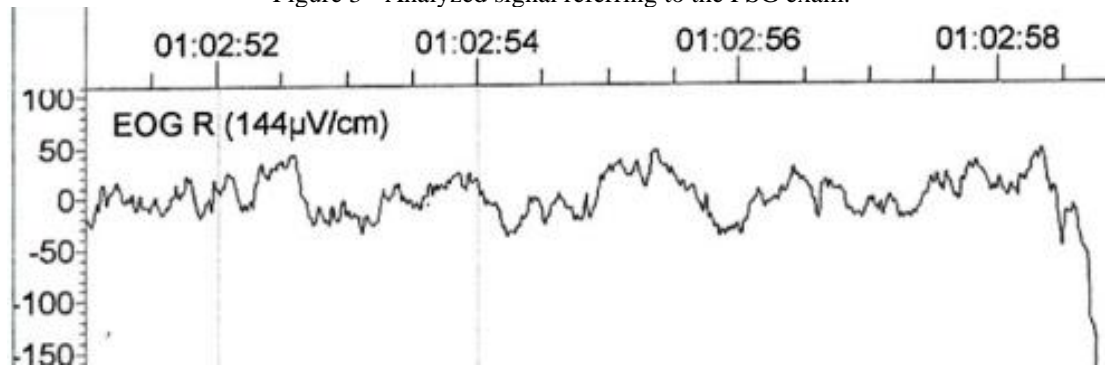
in which  $F(n)$  is the set of points representing the signal in time and  $N$  is the number of points sampled. Thus, to reduce the computational work, the technique called Fast Fourier Transform (FFT) is applied to execute (2). From there, the FFT algorithm divides the sequence  $x(n)$  into two smaller sequences, one with the even index coefficients and the other with the odd index coefficients. calculating the DFT for each of them and then combining them to obtain the DFT of the complete sequence (BRIGNHAM, 1974). It should be noted that for the FFT to be able to deliver a result with a clear definition of the frequencies present in the signal, a significant number of cycles are required during the execution of the algorithm. However, with each cycle increase, there is also an increase in time in the FFT calculation.

### 3.1 SIGNAL PROCESSING AND OBTAINING THE FREQUENCY SPECTRUM

Therefore, signals from a PSG exam were analyzed, as well as the sleep stages and their characteristics present in the exam, especially referring to deep sleep waves. In Figure 3, it is possible to observe the original signal of the PSG test. However, it was found that it has characteristics of Stage 3 of NREM sleep, referring to Delta waves, however, it also exposes attributes relevant to Stage 2, in which they are not the focus of the work. Thus, to carry out the signal processing, the original was considered and fragmented between the characteristics of Stage 2 and 3, in which the processed signal was obtained with the data only from Stage 3.

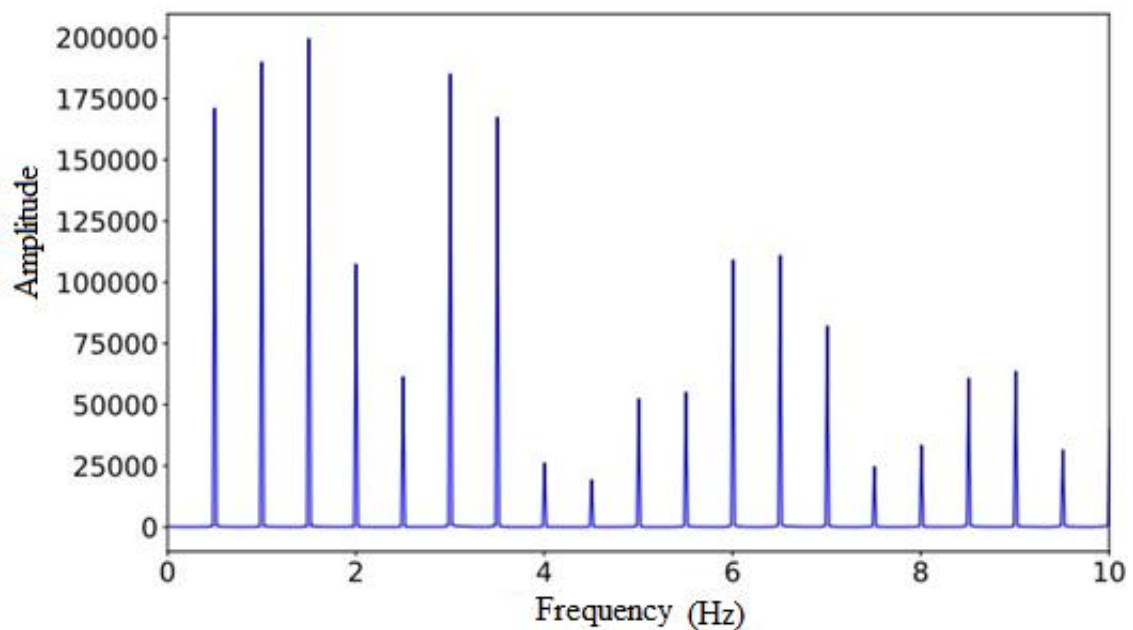


Figure 3 - Analyzed signal referring to the PSG exam.



The excerpt resulting from the processing of the PSG signal was used and, the Fast Fourier Transform (FFT) was applied with the aid of a python algorithm, with the purpose of obtaining the frequency spectrum referring to the signal. In order to obtain a more accurate result, the section under analysis was increased by 30 times. Based on this, the frequency spectrum shown in Figure 4 was obtained.

Figure 4 - Frequency spectrum referring to the processed signal.



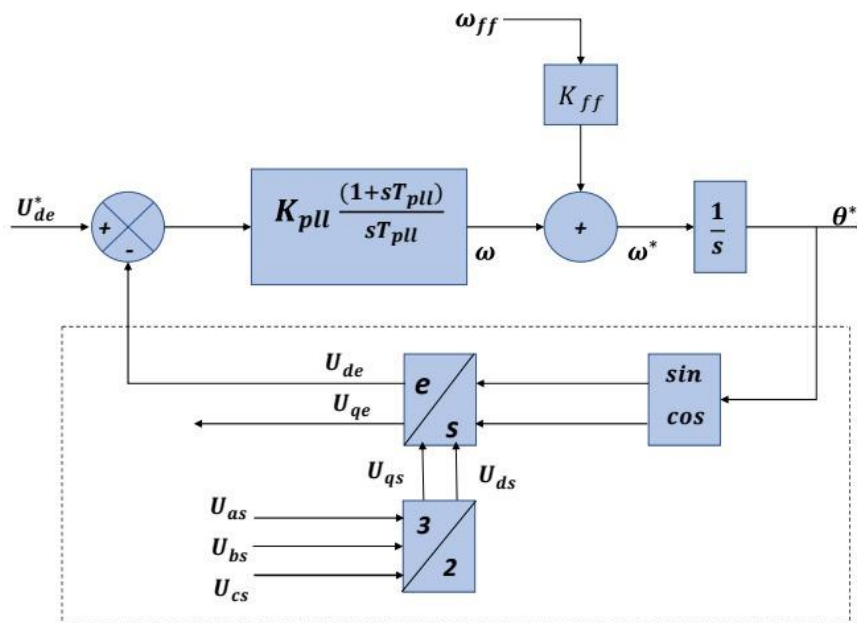
From Figure 4, it can be seen that the processed signal has high amplitudes for low frequencies, which confirms that it matches the characteristics of deep sleep, that is, referring to the Delta wave, in which it has its activity at a frequency of 0.5 to 4 Hz.

### 3.2 PHASE-LOCKED LOOP – PLL

The PLL structure is being increasingly used in a range of circuits that need power conditioning and that need to operate synchronously with the frequency of the electrical network (MARAFÃO, 2005). Therefore, because it is a response algorithm, it establishes the phase of the generated signal to adjust with the phase of a certain input signal (BOBROWSKA, 2011).

It has the function of estimating the frequency, amplitude and phase angle of an electrical signal, even with the presence of disturbances and harmonics, these being crucial information for the operation of several systems (KAURA, 1997). They can serve as protection, performance monitoring or be used for feedback, and are considered robust and easy to implement. Thus, one of the first modeled PLLs, the SRF-PLL, Synchronous Reference Frame, is demonstrated in Figure 5 (KAURA, 1997).

Figure 5 - SRF-PLL control diagram.



Thus, among the various techniques that the PLL can be applied, there is the FB-PLL - Fourier Based – Phase Locked Loop, which is equivalent to a selective perception of harmonics. FB-PLL refers to an algorithm that is suitable for single-phase and three-phase systems, which is based on the Fourier series, which is used as an instrument that grants the reconstruction of the main frequency of an input signal or some of its main frequencies. harmonic components (OPPENHEIM, 2009).



### 3.3 FOURIER BASED ALGORITHM

By means of an infinite sum of complex exponential functions, it is possible to obtain a periodic signal  $x(t)$  as long as some determinations are met (SANTOS, 2013):

- $x(t)$  be integrable in any period;
- $x(t)$  needs to have only limited variations, being over any period of time;
- $x(t)$  obtain only a finite number of discontinuities, being in any period of time.

Therefore, the Fourier series is described mathematically by the following equation:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t)) + (b_n \sin(n\omega_0 t)), \quad (3)$$

in which

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt, \quad (4)$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(n) dt, \quad (5)$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin(n) dt. \quad (6)$$

Thus,  $a_0$  represents the average value of  $x(t)$ , and  $a_n$  and  $b_n$  are equivalent to the amplitude of the cosine and sine elements of the  $n^\circ$  harmonic of the periodic signal  $x(t)$ . Based on this, the assumptions expressed by (5) and (6) are calculated as the average of the signal, multiplied by the frequency of interest. Therefore, the amplitude of the  $n^\circ$  harmonic is capable of being dynamically changed, this being through of the Low Pass Filter (LPF), which is the fundamental concept of the FB-PLL. In order to understand the





expanded FB-PLL principle, admit a signal  $v_s(t)$  that contains several harmonic components:

$$v_s(t) = \sum_{n=1}^N V_n \cos(n\omega t + \phi_n), \quad (7)$$

$V_n$  and  $\phi_n$  represent an amplitude and a phase of the component referring to  $n^\circ$  harmonic, respectively, and  $N$  equivalent to the harmonic element of greater order used to represent  $v_s(t)$ . It is assumed that the fundamental frequency of  $v_s(t)$  is known and that the amplitude and phase of each of the harmonic components are admitted. Initially, for the estimated harmonic,  $v_s(t)$  must be multiplied by two sinusoidal signals, both of which are present at the desired harmonic frequency, represented by:

$$v_{ic}(t) = \cos(H\omega t) \left( \sum_{n=1}^N V_n \cos(n\omega t + \phi_n) \right), \quad (8)$$

$$v_{is}(t) = \sin(H\omega t) \left( \sum_{n=1}^N V_n \cos(n\omega t + \phi_n) \right), \quad (9)$$

in which the harmonic order is represented by  $H$  and the subscripts  $i_c$  and  $i_s$  correspond to the integrand multiplied by the sine or cosine terms. By considering (8) and (9), and from the collection of terms of the desired harmonic order, as well as applying trigonometric properties, we have:

$$v_{ic} = \frac{aH}{2} (1 + \cos(2H\omega t)) + \frac{bH}{2} \sin(2H\omega t) + \cos(H\omega t) \left( \sum_{n=1}^N V_n \cos(n\omega t + \phi_n) \right), \quad (10)$$

$$v_{is} = \frac{bH}{2} (1 - \cos(2H\omega t)) + \frac{aH}{2} \sin(2H\omega t) + \sin(H\omega t) \left( \sum_{n=1}^N V_n \cos(n\omega t + \phi_n) \right), \quad (11)$$



In which  $1 \leq H \leq N$ . By examining (10) and (11) it is possible observe that  $v_{ic}(t)$  and  $v_{is}(t)$  have components of Direct Current, with magnitudes, respectively of  $\frac{aH}{2}$  and  $\frac{bH}{2}$ . The amplitudes  $a_H$  and  $b_H$ , referring to the  $v_{is}(t)$  and  $v_{ic}(t)$  signals can be acquired from a Low Pass Filter (LPF). These two signals can be divided into two segments, being Direct currente (DC) and Alternating Current (AC), as can be seen below (SANTOS, 2013).

$$v_{ic} = \bar{v}_{ic} + \tilde{v}_{ic}, \quad (12)$$

$$v_{is} = \bar{v}_{is} + \tilde{v}_{is}, \quad (13)$$

where  $v_{ic}$  corresponds to the DC component and  $\tilde{v}_{is}$  corresponds to the AC component. Thus,  $\bar{v}_{ic} = \frac{aH}{2}$  and  $\bar{v}_{is} = \frac{bH}{2}$ , therefore, by substituting in (12) and (13), we get:

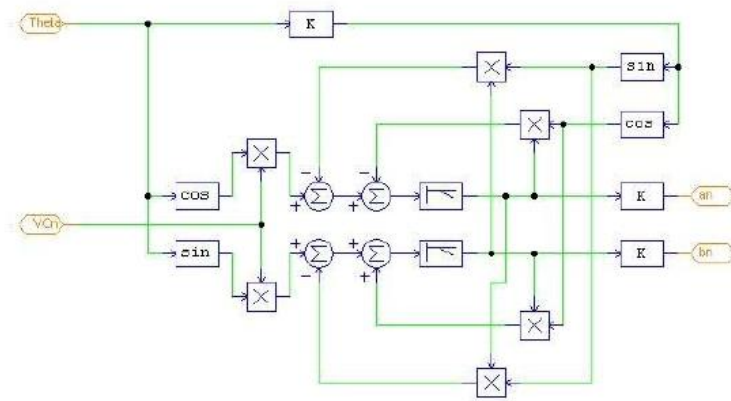
$$\tilde{v}_{ic} = \frac{aH}{2} \cos(2H\omega t) + \frac{bH}{2} \sin(2H\omega t) + \cos(H\omega t) \left( \sum_{n=1}^N V_n \cos(n\omega t + \phi_n) \right),$$

$$\tilde{v}_{is} = \frac{-bH}{2} \cos(2H\omega t) + \frac{bH}{2} \sin(2H\omega t) + \sin(H\omega t) \left( \sum_{n=1}^N V_n \cos(n\omega t + \phi_n) \right),$$

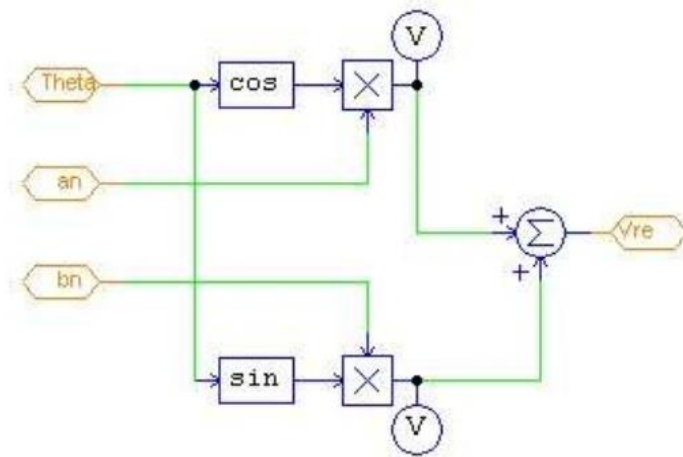
Thus, with (14) and (15) a structure called Dynamic Fourier Analysis (DFA) is disseminated, in which it is obtained through the reconstruction of the signals  $v_{ic}$  and  $v_{is}$ . It is used to improve filtering, reducing the amplitude of components at frequency  $2H\omega$ . Therefore, to obtain a clean entry in the DFA block, the harmonic components ( $n = 1, 2, \dots, H - 1, H + 1, \dots, N$ ) are eliminated, which makes the DFA is applied to each desired harmonic, and their values are estimated and subtracted [6]. A reform of the estimated signals takes place using the Original Signal Reconstruction (OSR) block. And, by placing a DFA block in series with the OSR block, the Dynamic Fourier System (DFS) is obtained. In this sense, Figure 6 (a) shows the DFA block, and Figure 6 (b) illustrates the OSR block used to build the desired harmonic with data from the amplitude and phase.



Figure 6 - (a) Dynamic Fourier analysis (DFA); (b) Original Signal Reconstruction (OSR).



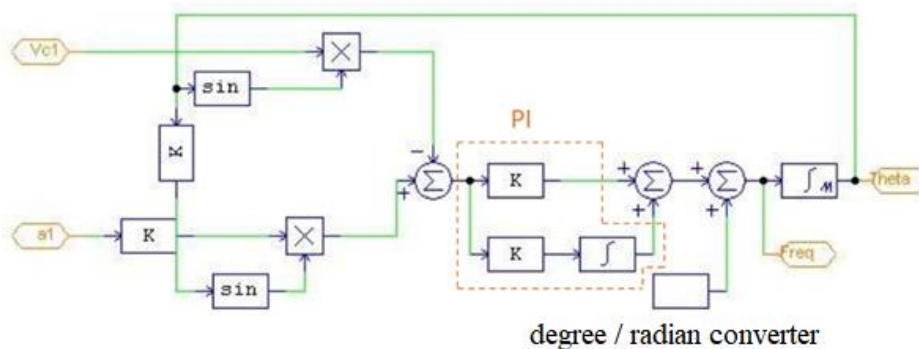
(a)



(b)

According to Figure 6, it is noticed that to obtain the desired frequency component, it is necessary to multiply sine and cosine. Thus, for the development of these two signals, phase information is needed, which are conceptualized using Phase Detector (PD), with its schematic portrayed by Figure 7.

Figure 7 - PD schematic diagram.





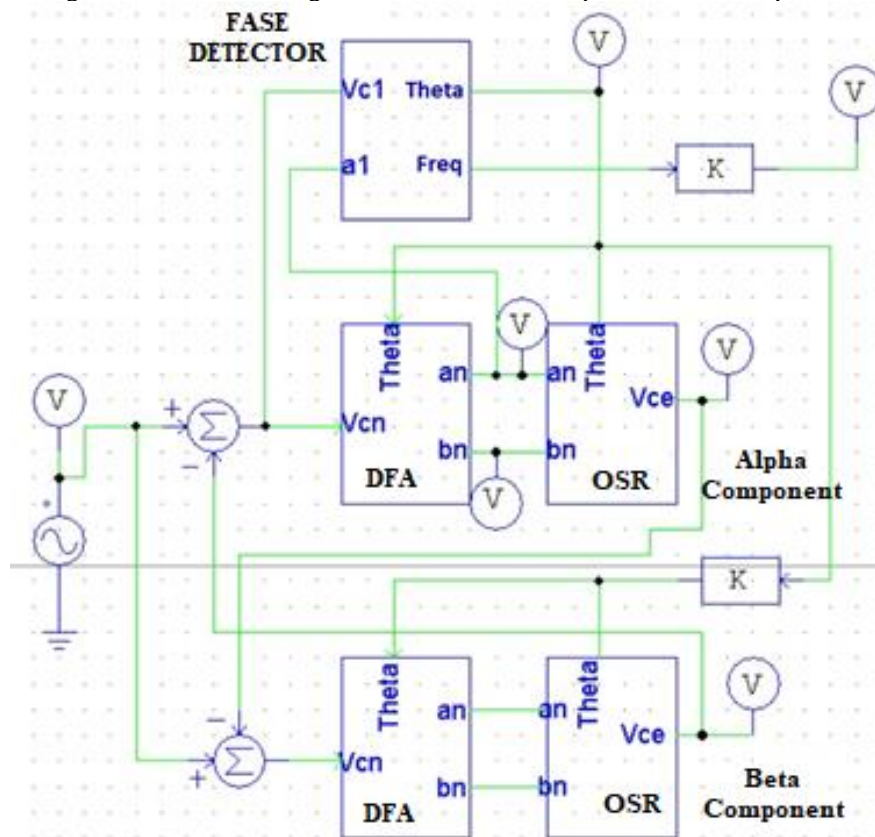
For the to Figure 7, it can be seen that the  $\theta$  phase is obtained, based on a Proportional-Integral (PI) Controller, in which it manipulates the error between the estimated signal and the input signal, in order to calculate the fundamental frequency. For the synthesis of the controller, the simulation itself was used as a reference, in which the gains were changed until an output with the desired characteristics was obtained. In order to evaluate the performance of the system according to the defined gains, the curve referring to the frequency was analyzed, as well as the phase signal  $\theta$  at the controller output. The integrative gain  $K_i$  and the proportional gain  $K_p$  can be seen in Table 1.

Table 1 - Gain values used in the PI controller referring to the PD block.

Gain	Values
$K_p$	180
$K_i$	80

A Fourier association was performed, which was produced to sum each OSR output, as shown in Figure 8.

Figura 8 - PLL block diagram based on Fourier Alpha and Beta components.



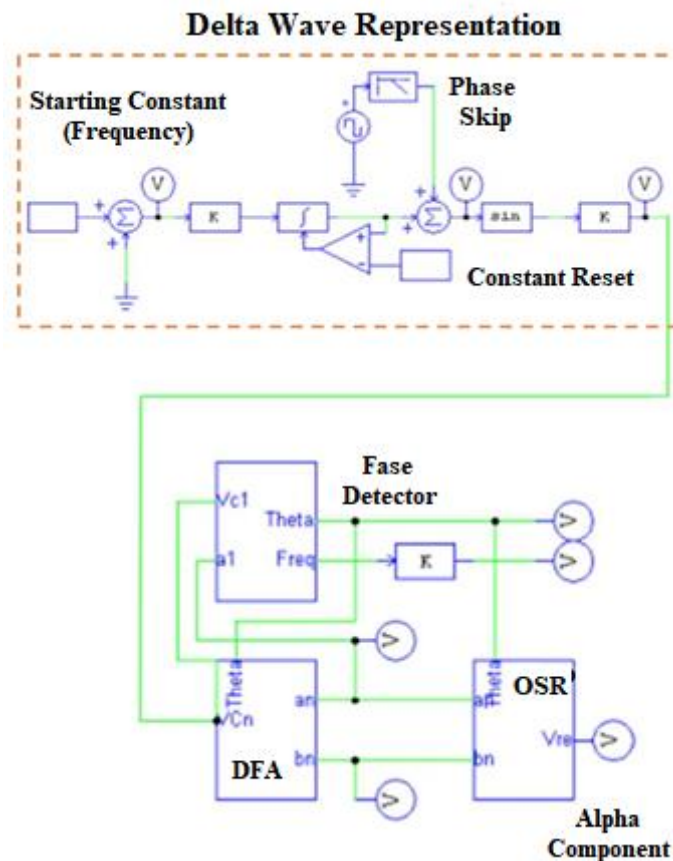
Also in Figure 8, it is possible to identify that each DFA and OSR pair is established for a harmonic. And, to obtain a clean signal in each DFA analysis, which is

called  $V_{cn}$  where  $n$  equals the harmonic order, it is necessary that the output of the OSRs be subtracted by the input signal. The FB-PLL was applied with only the 1<sup>o</sup> harmonic. Although other frequencies are present, these are not necessarily multiples of the fundamental, that is, they are not harmonic. Finally, it should be noted that the FB-PLL has to follow a signal with random variation in amplitude and frequency and with sporadic phase jumps. Also, it consists of a single-phase technique, suitable for an EEG signal.

#### 4 RESULTS AND DISCUSSIONS

Based on the FB-PLL circuit shown in Figure 8, the complete system was initially obtained, which can be analyzed in Figure 9.

Figure 9 - Complete system, with the FB-PLL block and a single-phase signal, symbolizing a wave referring to deep sleep, at the input  $V_{cn}$ .



From Figure 9, it can be seen that it formed an FB-PLL block, containing only the alpha component. This is because the beta component ends up zeroing out, that is, it becomes negligible for the simulation. Furthermore, it is possible to notice a new single-phase signal present at the input  $V_{cn}$ . Therefore, this signal consists of the representation of a wave present in Stage 3 of NREM sleep. Figures 10 and 11 show results referring to



the FB-PLL simulation applied to a signal with typical Delta wave behavior, which in the figure corresponds to the original signal, with an amplitude of approximately  $180 \mu V$ . In all simulations, random variations of amplitude, frequency and phase are applied, but at some points the frequency varies abruptly between 2 and 4 Hz.

Figure 10 - Simulation Result of Estimation of Amplitude and Frequency of a Delta Wave Signal: decrease in Frequency.

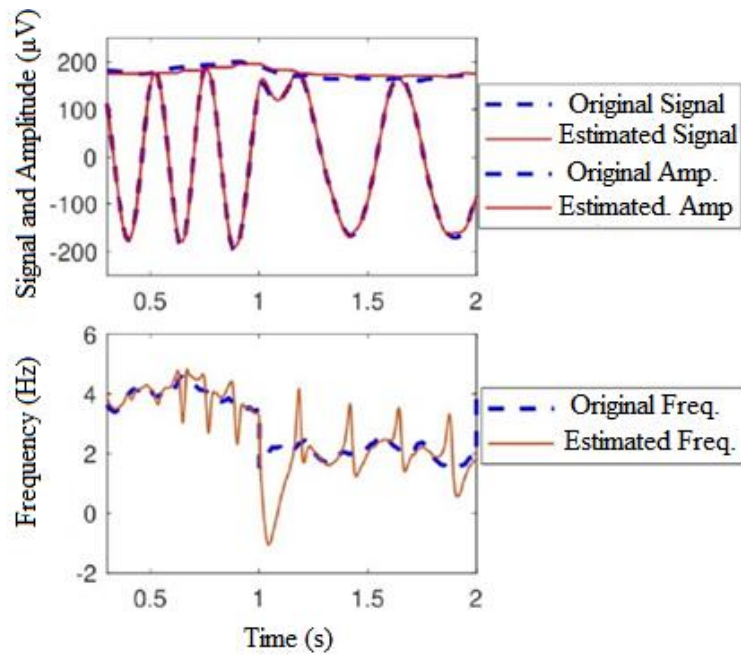


Figure 11- Simulation Result of Estimating the Amplitude and Frequency of a Delta Wave Signal: frequency Increase.

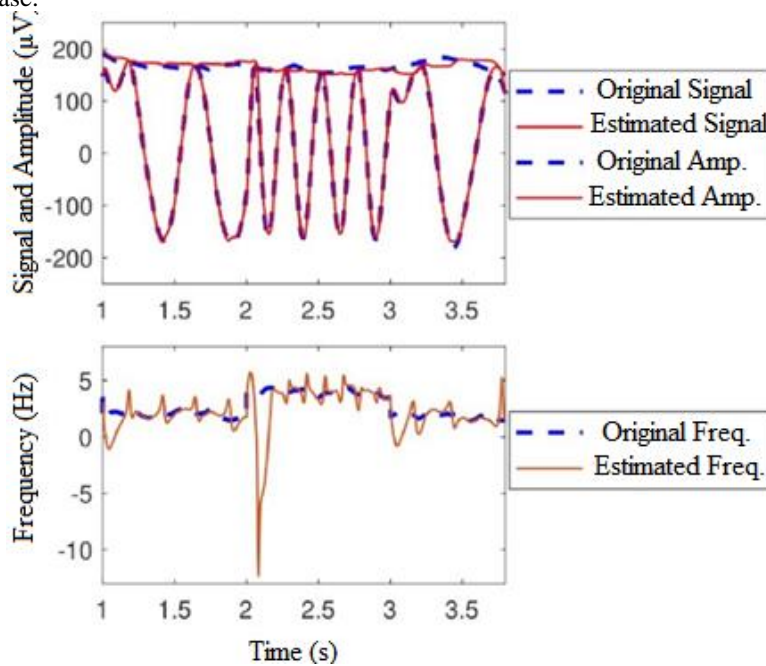




Figure 10 (a) shows the performance of the FB-PLL against a frequency decrease of approximately 4 Hz to values close to 2 Hz, as can be seen in Figure 10 (b). Thus, it is noted that the estimated frequency follows the actual frequency of the signal in most of the simulation range. Even with a phase jump at instant 1 second of the simulation, the estimated frequency continues to follow the input frequency. It can also be observed that, in Figure 10 (a), the estimated amplitude follows the reference amplitude, even with random variations. The estimated signal is built with the estimated amplitude, frequency and phase information, and as observed, this signal is very close to the original delta wave signal.

In Figure 11 (a) it is possible to observe the behavior of the FB-PLL in relation to the increase in frequency from about 2 Hz to approximately 4 Hz, as can be seen in Figure 11 (b). Thus, it can be observed that the estimated frequency can also follow the input frequency in a good time of the simulation, similarly when the frequency decreases. Furthermore, in Figure 11 (a), it is noted that the estimated amplitude remains following the input amplitude, including the application of perturbations during the simulation. In relation to the estimated signal, it is noticed that it approaches the original signal.

It appears that the FB-PLL is able to estimate, most of the time, a change in frequency through an input signal, which has varying amplitude and phase jumps. Furthermore, it is inferred that the FB-PLL technique is able to estimate the signal both at low and high frequencies. Therefore, this is of paramount importance for the scope of polysomnography, as it is thus possible to identify the frequency during sleep and, consequently, the Stage in which the person is.

## 5 CONCLUSION

A low frequency wave sleep is essential for maintaining mental and physical health, in which it cooperates directly with the promotion of the quality of life of human beings. In this sense, it is important to monitor brain waves during sleep, and to know the frequency measured during sleep.

The FFT technique is able to estimate the frequency of these signals. However, to obtain an accurate result, a considerable number of sampling cycles is essential. In addition, with the addition of so many, a significant amount of time is required to calculate the FFT, which makes it difficult to use it to perform a real-time reading. The FB-PLL method, on the other hand, manages to carry out the estimates in just one cycle, contributing to having a result in a short period of time.



Applying the FB-PLL technique to a signal with typical Delta wave behavior, it was possible to notice that when there is a reduction in frequency from 4 Hz to 2 Hz, as well as during an increase from 2 Hz to 4 Hz, it was noted that the estimated frequency continued to follow the real one. Furthermore, it is important to point out that even with a phase jump, the estimated frequency continues to follow the input frequency. It is also inferred that, in both cases, the estimated amplitude follows the reference amplitude, even with random variations. The estimated signal was very close to the original signal.

Therefore, it is concluded that the FB-PLL was able to estimate, most of the time, a frequency variation via an input signal, in which it has varying amplitude and phase jumps. Furthermore, it is believed that the technique is able to estimate the signal both at low and at high frequencies. Thus, this is of great relevance for the sphere of Sleep Medicine, since it recognizes the frequency and, consequently, the Stage in which the person is. This contributes to the diagnosis of a range of diseases and disorders related to sleep aptitude, which will be addressed in future work.





## REFERÊNCIAS

ABEL; T. HAVEKES; R. SALETIN; J. WALKER, M. P. Sleep, plasticity and memory from molecules to whole-brain networks. *Current Biology: CB*, v. 23, n. 17, p. R774-88, 2013.

GOLDSTEIN-PIEKARSKI, A. N; GREER, S. M; SALETIN, J. M; WALKER, M. P. Sleep deprivation impairs the human central and peripheral nervous system discrimination of social threat. *The Journal of Neuroscience*, v. 35, n. 28, 2015.

PEREIRA, I. R; FONSECA, T. R; SANTOS, C. H. G. Review of the main aspects of brain waves related to sleep. *Brazilian Interdisciplinary Congress on Science and Technology*, São Paulo, p. 1-9, 2023, in press.

WILCKENS, K. A; FERRARELLI, F; WALKER, M. P; BUYASSE, D. j. Slow-wave activity enhancement to improve cognition. *Trends in Eurosciences*, v. 41, n. 7, p. 470–482, 201.

WALKER, M. P. The role of slow wave sleep in memory processing. *Journal of Clinical Sleep Medicine: JCSM: official publication of the american academy of sleep medicine*, v. 5, n. 2 Suppl, p. S20-6, 2009.

ASTILL, R. G. et al. Sleep spindle and slow wave frequency reflect motor skill performance in primary schoolage children. *Frontiers in Human Neuroscience*, v. 8, p. 910, 2014.

SPIEGEL, M. R. *Fourier analysis with applications to boundary value problems*. Mc Graw Hill, New York, 1974.

BRIGNHAM, E. *The fast Fourier transform*. Prentice Hall, New Jersey, 1974.

MARAFÃO, Fernando P. et al. DESIGN METHODOLOGY AND ANALYSIS OF PLL SYNCHRONISM ALGORITHMS. *Power Electronics*, vol. 10, no. 1, 2005.

BOBROWSKA, M.R; RAFAL, K; JASINSKI, M; KAZMIERKOWSKI, M.P. Grid synchronization and symmetrical components extraction with PLL algorithm for grid connected power electronic converters – a review. *Bulletin of the Polish Academy of Sciences Technical Sciences*, v. 59, n. 4, 2011.

KAURA, V; BLASKO, V. Operation of a phase locked loop system under distorted utility conditions. *IEEE Transactions on Industry Applications*, v. 33, n. 1, 1997.

OPPENHEIM, A. V.; WILLSKY, A. S. *Signals and systems*. 2nd. New Delhi: Prentice Hall, 2009.

SANTOS, C. H. G; FERREIRA, R; SILVA, S. M; FILHO, B. C. Fourier-based PLL applied for selective harmonic estimation in electric power systems. *Journal of Power Electronics*, v. 13, n. 5, 2013. DOI: 10.6113/JPE.2013.13.5.884.